Calculus Problem List

*Problems are categorized by exam, though subject material may vary between courses

Exam 1

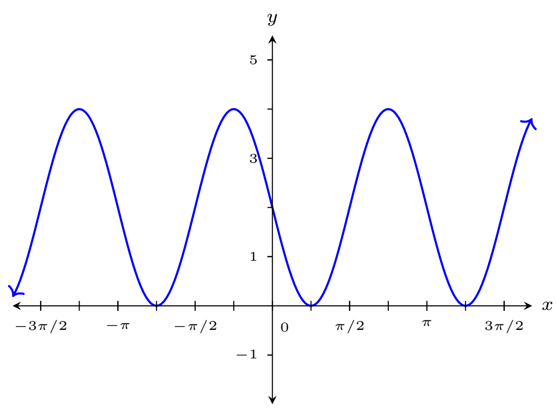
1. Find the slope and y-intercept of the line $3y - 4 = 5x + 2$.

2. If a page is reduced to 75%, what percentage enlargement is required to return it to its original size?

3. Find the inverse of the function $f(x) = \dfrac{x+2}{x-3}$.

4. Solve for x: $2e^{3x} = 6e^{2x}$

5. Determine an equation for the sinusoidal function below:



6. A pomegranate is thrown from ground level straight up into the air at time $t = 0$ with velocity 240 feet per second. Its height in feet at t seconds is $f(t) = -16t^2 + 240t$. Find the time it hits the ground and the time it reaches its highest point.

7. Find k so that the following function is continuous on any interval:

$$f(x) = \begin{cases} kx & x < 2 \\ 9x^2 & x \geq 2 \end{cases}$$

8. Evaluate the following limit. If the limit does not exist enter DNE.

$$\lim_{x \to 6} \frac{x^2 - 2x - 24}{x - 6}$$

9. Find the derivative of the following function using the limit definition of derivative. (Note: you will not receive any credit for simply applying a law of differentiation. You must use the limit definition)

$$g(x) = \sqrt{2x - 1}$$

10. (a) Given that $g(3) = 3/2$ and $g'(3) = \pi$ construct the tangent line of $g(x)$ at $x = 3$
    (b) Using the fact that $g''(x) < 0$ for all x, what are the possible values for $g(6)$? Why

11. Given $f(x) = \frac{3x+2}{x-1}$, state the domain and range then find $f^{-1}(x)$.

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12. Let $f(x) = \ln(2x-3)$, state the domain and range then find $f^{-1}(x)$.

13. $5e^{2x-1} + \frac{1}{2} = 2$

14. $\ln(\frac{1}{x}) + \ln(2x^2) = \ln(3)$

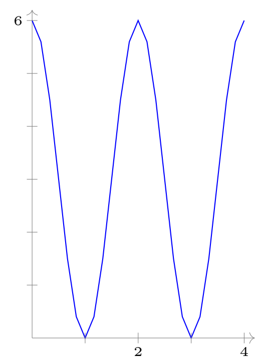
15. Find the derivative of the following functions using the limits definition of derivative. (Note: you will not receive any credit for simply applying a law of differentiation. You must use the limit definition)

    (a) $g(x) = \sqrt{x+2}$

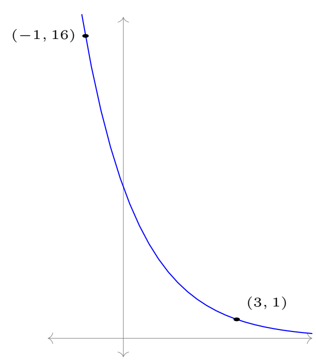
    (b) $h(x) = (x-3)^{-1}$

16. Find a possible formula for the following graphs

    (a) Sinusoidal function



    (b) Exponential function



17. Let $f(x) = \frac{1}{x^2}$ and $g(x) = kx^2$ where k is constant.

    (a) Find the slope of the tangent line to the graph of $f(x)$ at $x = 1$ (You may use the power rule).

    (b) Find the equation of the tangent line to $f(x)$ at $x = 1$.

    (c) The graph of g contains the point $(1, 1)$, find the value of k.

    (d) At which two points does the graph of $g(x)$ intersect the tangent line found in part b.

18. Let $P(t)$ be the population of China in millions of people and t be measured in years since 2000.

    (a) Explain the meaning of $f(0) = 1,263$, make sure to include units.

(b) Explain the meaning of $f^{-1}(1,338) = 10$, make sure to include units.

(c) Explain the meaning of $f'(5) = 41$, make sure to include units.

19. Find the values m and b such that the following function is continuous.

$$f(x) = \begin{cases} 2e^{x+2} & x \leq -2 \\ mx + b & -2 < x < 1 \\ 2\sqrt{x+3} & x > 1 \end{cases}$$

20. Compute the following limits if they exist, if they do not exists write DNE.

(a) $\lim_{x\to 3} \frac{x^2-2x}{x+1}$

(b) $\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4}$

(c) $\lim_{x\to 1^-} \frac{2x}{x-1}$

(d) $\lim_{x\to -\infty} \frac{2x^4+3x^2-2}{\pi(1+x^4)}$

21. Find the derivative of the following functions using the limits definition of derivative. (Note: you will not receive any credit for simply applying a law of differentiation. You must use the limit definition)

(a) $g(x) = \sqrt{x+2}$

(b) $h(x) = (x-3)^{-1}$

22. (a) Given that $g(3) = 3/2$ and $g'(3) = \pi$ construct the tangent line of $g(x)$ at $x = 3$

(b) Using the fact that $g''(x) < 0$ for all x, what are the possible values for $g(6)$? Why?

Exam 2

23. A particle's position (in meters) on the x-axis is given by the following table. Estimate the instantaneous velocity of the particle at time 1.2 s and 1.4 s.

t	1.0	1.2	1.4	1.6
$s(t)$	4	0	-3	8

24. Suppose that $f(x)$ is a function with $f(90) = 70$ and $f'(90) = 7$. Estimate $f(94)$.

25. Use algebra to evaluate the limit $\lim_{h\to 0} \frac{(5+h)^3 - 125}{h}$.

26. Find $f'(x)$ if $f(x) = 3x^2 - 7x + 40329$.

27. The cost, C (in dollars) to produce g gallons of icecream can be expressed as $C = f(g)$. If $f(225) = 325$ and $f'(225) = 1.8$,
what are the units of 225?

what are the units of 325?

what are the units of 1.8?

Calculus Problem List

28. Is the function $f(x) = |x-1|$ differentiable everywhere? Explain why or why not.

29. Evaluate the following derivatives:

(a) $\frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ (b) $\frac{d}{dx}\left(17^x\right)$

30. Find the intervals where the function $f(x) = x^3 + \frac{15}{2}x^2 + 18x - 1$ is increasing and decreasing. State where the function has a max or a min.

31. Calculate the following derivatives:

(a) $\frac{d}{dx}(x^2 4^x)$ (b) $\frac{d}{dx}(x \sin(\frac{1}{x}))$ (c) $\frac{d}{dx}(\ln(x)\arctan(x))$

32. Calculate the following derivatives:

(a) $\frac{d}{dx}(\arcsin 5t + 10)^{30}$ (b) $\frac{d}{dx}\left(\frac{\sin(4x)}{\cos(6x)}\right)$ (c) $\frac{d}{dx}(\sqrt{x + \sqrt{x}})$

33. Find the max, min, intervals of increasing, decreasing, concave up, and concave down for the function $f(x) = x\ln(x)$.

34. Use the table to calculate the following:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	-4	9	4	1
1	2	2	7	-3
2	-1	6	-4	3

(a) $\frac{d}{dx}(f(x)g(x))\Big|_{x=0}$ (b) $\frac{d}{dx}(f(g(x)))\Big|_{x=1}$ (c) $\frac{d}{dx}(f^{-1}(x))\Big|_{x=2}$

35. (a) Find the local linear approximation of $y = \sqrt[3]{x-4}$ at $x = 12$.

(b) Use your result to approximate $\sqrt[3]{9}$.

36. Calculate the following derivatives. Use limit definitions, derivative rules, implicit differentiation, or the second fundamental theorem of calculus:

(a) $\frac{d}{dx}\left(2^x \ln(x)\right)$

(b) $\frac{d}{dx}\left(x^x\right)$

(c) $\frac{d}{dx}\int_{3x^2}^{\ln(x)} \tan(x)\,dx$

(d) $\frac{d}{dx}\int_{-15}^{x^2-2x} \frac{1}{x}\,dx$

37. Given that $h(2) = 5$ and $h'(2) = 3$, if $f(x) = h^{-1}(x)$, find $f'(5)$. No credit will be given if no work is shown.

38. Find and classify all critical points of the function $f(x) = x^2e^{-3x}$. Find the intervals where $f(x)$ is increasing and decreasing.

Calculus Problem List

39. Find values of a and b so that the function

$$f(x) = \begin{cases} x^2, & -\infty \leq x \leq 2 \\ -(x-a)^2 + b, & 2 \leq x \leq \infty \end{cases}$$

is both continuous and differentiable everywhere.

40. Let $g(x) = \frac{6-x}{2x+1}$ then

   (a) Find the derivative at the value $x = 1$.

   (b) Construct the equation of the tangent line at the value $x = 1$.

   (c) Construct the equation of the line perpendicular to the tangent line at $x = 1$.

41. Given the following values

x	-2	0	2	4	6
$f(x)$	-3	-1	2	0	-2
$g(x)$	5	2	-3	2	0
$f'(x)$	-2	2	4	-1	0
$g'(x)$	-3	-1	2	3	4

   (a) If $h_1(x) = f \circ g(x)$ find $h_1'(0)$.

   (b) If $h_2(x) = f(x)g(x)$ find $h_2'(2)$.

   (c) If $h_3(x) = \frac{f(x)}{g(x)}$ find $h_3'(-2)$.

   (d) If $h_4(x) = 2 \cdot f^{-1}(x)$ find $h_4'(0)$.

42. Find the derivatives to these implicit functions.

   (a) $x \ln(y) + y^2 = \ln(x)$

   (b) $e^{xy} = 2$

   (a) Find the local linear approximation of $y = \sqrt[3]{x}$ at $x = 8$.

   (b) Use your result to approximate $\sqrt[3]{9}$.

   (c) Is this an over or under estimate? (Show work here)

43. Let $f(x) = x \ln(x) + x$, on the interval $(0, \infty)$.

   (a) Find the interval(s) where $f(x)$ is increasing.

   (b) Find the interval(s) where $f(x)$ is concave up.

44. Evaluate the following trigonometric functions

   (a) $\cos(5\pi/3)$

   (b) $\tan(5\pi/6)$

   (c) $\sin^{-1}(\frac{1}{2})$

   (d) $\tan^{-1}(-1)$

45. Find the derivatives for the following functions

   (a) $y = \ln(\pi^x)$

(b) $y = \frac{e^x}{x^3+3x^2+2x+1}$

(c) $y = \arctan(2x)$

(d) $y = \sqrt{2^x \cdot \sin(x)}$

46. Let $f(x) = e^{\frac{x^3}{3} - \frac{x^2}{2} - 6x + 2}$ then

    (a) Find $f'(x)$.

    (b) For which x-values is $f'(x) = 0$?

    (c) Find the interval(s) for which $f(x)$ increasing.

47. Let $g(x) = \frac{5-x}{2x+1}$ then

    (a) Find the derivative at the value $x = 1$.

    (b) Construct the equation of the tangent line at the value $x = 1$.

    (c) Construct the equation of the line perpendicular to the tangent line at $x = 1$.

48. Find the derivatives to these implicit functions.

    (a) $x\ln(y) + y^2 = \ln(x)$

    (b) $e^{xy} = 2$

49. Given the following values

x	-2	0	2	4	6
$f(x)$	-3	-1	2	0	-2
$g(x)$	5	2	-3	2	0
$f'(x)$	-2	2	4	-1	0
$g'(x)$	-3	-1	2	3	4

    (a) If $h_1(x) = f \circ g(x)$ find $h_1'(0)$.

    (b) If $h_2(x) = f(x)g(x)$ find $h_2'(2)$.

    (c) If $h_3(x) = \frac{f(x)}{g(x)}$ find $h_3'(-2)$.

    (d) If $h_4(x) = 2 \cdot f^{-1}(x)$ find $h_4'(0)$.

50. (a) Find the local linear approximation of $y = \sqrt[3]{x}$ at $x = 8$.

    (b) Use your result to approximate $\sqrt[3]{9}$.

    (c) Is this an over or under estimate? (Show work here)

51. Construct the equation of the tangent line for the following curve at the point $(1, \pi/2)$,

$$\cos(xy) + x^2 = 1.$$

52. Let $f(x) = x\ln(x) + x$, on the interval $(0, \infty)$.

    (a) Find the interval(s) where $f(x)$ is increasing.

    (b) Find the interval(s) where $f(x)$ is concave up.

Calculus Problem List

Exam 3

53. (a) Find the tangent line approximation to $\sqrt{x+2}$ at $x = 2$.
    (b) Use the tangent line to approximate $\sqrt{5}$.
    (c) Estimate the error for your approximation in part (b).
54. (a) State precisely the Mean Value Theorem
    (b) Does the function $f(x) = \ln(x+3)$ satisfy the hypothesis of the Mean Value Theorem on the interval [-1,1]? If so find the value of c. If not why not?
55. Find the exact global maximum and minimum values of the function $g(t) = 6te^{-8t}$ if $t \geq 0$.
56. Find the equation of a quartic polynomial whose graph is symmetric about the y-axis and has local maxima at $(-1, 8)$ and $(1, 8)$ and a y-intercept of 5. ‘
57. A rectangular box with no top is to be made out of 120cm^2 of cardboard. Find the dimensions of the box with the maximum volume. What is the maximum volume?
58. A glass fish tank is to be constructed to hold 80ft^3 of water. The top, of course, is to be open. It is to be constructed so that its width will be 5 ft but the length and depth are variable. Building the tank costs $10 per square foot for the base and $5 per square foot for the sides. What is the cost of the least expensive tank? Justify your answer.
59. Use L'Hopitals Rule to calculate the following limit.:
$$\lim_{x \to 0} \frac{x}{e^x - 1}$$
60. Gravel is being dumped from a conveyor belt at a rate of 10 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 18 feet high? Recall that the volume of a right circular cone with height h and radius of the base r is given by $V = \frac{1}{3}\pi r^2 h$.
61. Show that the function $f(x) = \frac{x}{x+1}$ satisfies the Mean Value Theorem on $[0, 2]$ by finding the value c.
62. Calculate the following limits:

    (a) $\lim_{x \to 1} \frac{(x-1)^2}{\ln x}$

    (b) $\lim_{x \to 0} (1 + 2\sin(x))^{\frac{1}{x}}$
63. Suppose that the position of a particle at time t is given by $x(t) = \sin(2t)$ and $y(t) = \cos(2t)$.
    (a) What is the shape of the particle's path?
    (b) Find $\frac{dy}{dx}$ when $t = 5\pi/6$.
64. A 25 ft ladder is leaning against a wall. The floor is slightly slippery and the foot of the ladder slips away from the wall at a rate of 3 ft/s. How fast is the top of the ladder sliding down the wall when the base is 20 ft from the wall?
65. Decide if the statement is true or false and give an explaination for your answer: The Racetrack Principle can be used to justify the statement that if two horses start a race at the same time, the horse that wins must have been moving faster than the other throughout the race.

Calculus Problem List

66. Let $p(x)$ be a seventh degree polynomial with 7 distinct zeros. How many zeros does $p'(x)$ have? Justify your answer.

67. Sketch a possible graph of $y = f(x)$ using information about the derivatives. Assume that the function is defined and continuous for all real x.

68. Find the global maximum and minimum for the function on the closed interval.

$$f(x) = x^3 - 3x^2 + 20, \quad -1 \leq x \leq 3$$

69. Find the global maximum and minimum for the function on the closed interval.

$$f(x) = x - 2\ln(x+1), \quad 0 \leq x \leq 2$$

70. Find a formula for the family of cubic polynomials with an inflection point at the origin. How many parameters are there?

71. Find the x-value maximizing the shaded area. One vertex is on the graph of $f(x) = x^2/3 - 50x + 1000$.

72. A rectangle has one side on the x-axis and two vertices on the curve

$$y = \frac{1}{(1+x^2)}.$$

Find the vertices of the rectangle with maximum area.

73. When production is 2000, marginal revenue is \$4 per unit and marginal cost is \$3.25 per unit. Do you expect maximum profit to occur at a production level above or below 2000? Explain.

74. Let $C(q)$ be the total cost of producing a quantity q of a certain product. See Figure 4.76.
(a) What is the meaning of $C(0)$?
(b) Explain the shape of the curve in terms of economics.

75. Gasoline is pouring into a cylindrical tank of radius 3 ft. When the depth of the gasoline is 4 ft, the depth is increasing at 0.2 ft/sec. How fast is the volume of gasoline changing at that instant?

76. The radius of a spherical balloon is increasing by 2 cm/sec. At what rate is air being blown into the balloon at the moment when the radius is 10 cm? Give units in your answer.

77. Evaluate the following limit:

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

78. Evaluate the following limit:

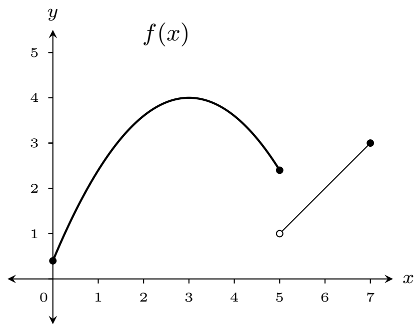
$$\lim_{x \to 0^+} x^x$$

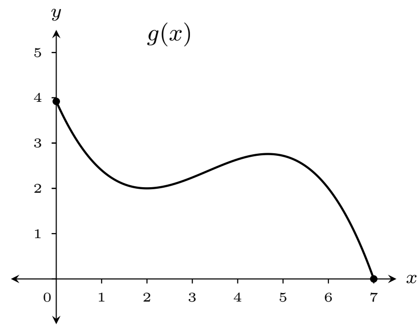
(a) State precisely the Mean Value Theorem.

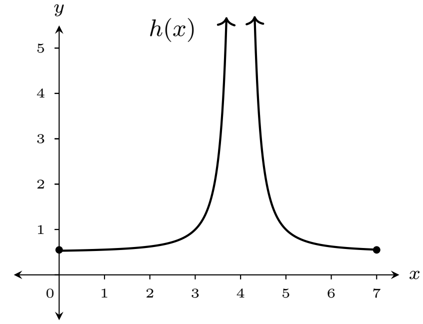
(b) Does the function $f(x) = \ln(x+1)$ satisfy the hypothesis of the Mean Value Theorem on the interval $[1, e-2]$? Explain.

(c) Show that the function $f(x) = \ln(x+2)$ satisfies the conclusion of the Mean Value Theorem on $[0, e-1]$ by finding the value c.

79. (b) Which of the following functions satisfies the hypothesis of the Mean Value Theorem on [0,7]?
    (c) Which functions satisfy the conclusion?

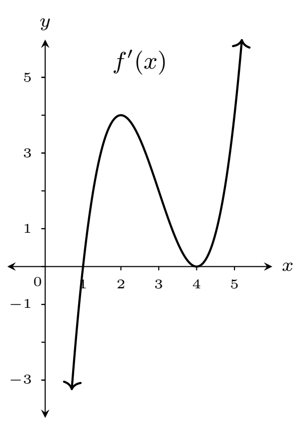






4.1

80. (a) Given the graph of $f'(x)$ find and classify all critical points of $f(x)$.
    (b) Does $f(x)$ have a global max or global min on $(-\infty, \infty)$? Explain, providing the location if it exists.



81. Find and classify all critical points of the function $f(x) = \dfrac{xe^{-x^2}}{x-3}$.
    (comment: *A local max and local min exists as well as a critical point at x=3 where the derivative is undefined.*)

82. Find and classify all critial points of the function $f(x) = x^2(x-2)^{\frac{1}{3}}$.
    (comment: *A local max and local min as well as a critical point at $x = 2$ where the derivative is undefined.*)

83. Use the Racetrack Principle to show that $\ln x \leq x - 1$ for all $x \geq 1$.

84. Decide if the statement is true or false and give an explaination for your answer: The Racetrack Principle can be used to justify the statement that if two horses start a race at the same time, the horse that wins must have been moving faster than the other throughout the race.

85. Let $p(x)$ be a seventh degree polynomial with 7 distinct zeros. How many zeros does $p'(x)$ have? Justify your answer.

Calculus Problem List

86. Sketch a possible graph of $y = f(x)$ using information about the derivatives. Assume that the function is defined and continuous for all real x.

87. Find the global maximum and minimum for the function on the closed interval.

$$f(x) = x^3 - 3x^2 + 20, \quad -1 \leq x \leq 3$$

88. Find the global maximum and minimum for the function on the closed interval.

$$f(x) = x - 2\ln(x+1), \quad 0 \leq x \leq 2$$

89. Find a formula for the family of cubic polynomials with an inflection point at the origin. How many parameters are there?

90. Find and classify all critical points for the following functions:
    (a) $f(x) = x^3 - x^2 - 5x + 7$
    (b) $f(x) = e^{x^2-4x}$

91. Find and classify all critical points of the function $f(x) = x^2 e^{-3x}$

92. Find the global maximum and global minimum of the function $f(x) = 6xe^{-x/2}$ on the interval $[0, \infty)$

93. Find a formula for the function described: a cubic polynomial with a local maximum at $x = -2$, a local minimum at $x = 1$, a y-intercept of -2, and an x^3-term whose coefficient is 1.

94. A rectangle has one side on the x-axis and two vertices on the curve $e^{-x^2/6}$. What x-value gives the maximum area? What is the maximum area?

95. A rectangle has one side on the x-axis and two vertices on the curve

$$y = \frac{1}{(1+x^2)}.$$

Find the vertices of the rectangle with maximum area.

96. When production is 2000, marginal revenue is \$4 per unit and marginal cost is \$3.25 per unit. Do you expect maximum profit to occur at a production level above or below 2000? Explain.

97. Let $C(q)$ be the total cost of producing a quantity q of a certain product. See Figure 4.76.
    (a) What is the meaning of $C(0)$?
    (b) Explain the shape of the curve in terms of economics.

98. Gasoline is pouring into a cylindrical tank of radius 3 ft. When the depth of the gasoline is 4 ft, the depth is increasing at 0.2 ft/sec. How fast is the volume of gasoline changing at that instant?

99. The radius of a spherical balloon is increasing by 2 cm/sec. At what rate is air being blown into the balloon at the moment when the radius is 10 cm? Give units in your answer.

100. Evaluate the following limit:

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

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101. Evaluate the following limit:
$$\lim_{x \to 0^+} x^x$$

102. A Landscape architect plans to enclose a 125 square foot rectangular region in a botanical garden. She will use wire fencing that costs \$8 per foot along three sides and wooden fencing that costs \$12 along the fourth side. Find the minimum total cost.

103. Suppose you are selling q units of a product with a cost function of $C(q)$ and a revenue function of $R(q)$. If $C'(300) = 120$ and $R'(300) = 90$, should the quantity produced be increased or decreased from $q = 300$ in order to increase profits? Explain.

104. The total cost $C(q)$ and total revenue $R(q)$ of producing q goods are given by the equations:
$$C(q) = 20 + 10q \qquad R(q) = 50q - q^2$$
Write a function that gives total profit earned and find the quantity that maximizes profit.

105. A 25 ft ladder is leaning against a wall. The floor is slightly slippery and the foot of the ladder slips away from the wall at a rate of 3 in/s. How fast is the top of the ladder sliding down the wall when the base is 20 ft from the wall?

106. A spherical snowball is melting at a rate of 6 cm^3/min. How fast is the radius of the snowball decreasing when its radius is 18 cm?

107. For the amusement of guests, some hotels have elevators on the outside of the building. One such hotel is 300 feet high. You are standing by a window 100 feet above the ground and 150 feet away from the hotel, and the elevator descends at a constant speed of 25 ft/sec, starting at time $t = 0$, where t is in seconds. Let θ be the angle between the line of your horizon and your line of sight to the elevator. How fast is the angle θ changing after 6 seconds?

108. Calculate the following limits:
    - $\lim_{x\to\infty} \frac{e^x}{x^2}$
    - $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{\cos x - 1}$
    - $\lim_{x\to\infty} e^{-x} \ln x$
    - $\lim_{x\to 0} (1+7x)^{\frac{1}{x}}$

109. Suppose that the position of a particle at time t is given by $x(t) = -3t + e^t$ and $y(t) = 4t - 4$.
(a) Find the position of the particle at time $t = 0$.
(b) Find $\frac{dy}{dx}$ when $t = 0$.

Final Exam

110. A car comes to a stop six seconds after the driver applies the brakes. While the brakes are on, the following velocities are recorded:

Time since brakes applied (sec)	0	2	4	6
Velocity (ft/s)	97	49	18	0

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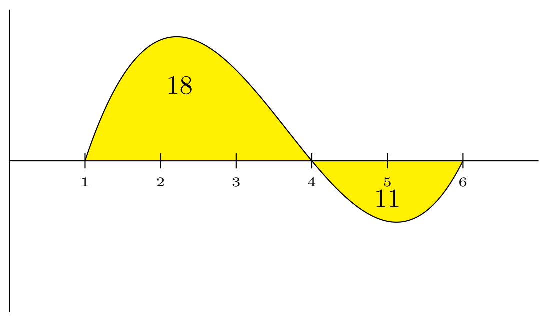
Give left and right estimates (using Δt = 2) for the distance the car traveled after the brakes were applied.

111. Find the average value of $f(x) = 8x + 3$ over $[5, 7]$.

112. Use the following figure, which shows a graph of $f(x)$ to find each of the indicated integrals.



(a) $\displaystyle\int_1^4 f(x)\,dx$

(b) $\displaystyle\int_4^6 f(x)\,dx$

(c) $\displaystyle\int_1^6 f(x)\,dx$

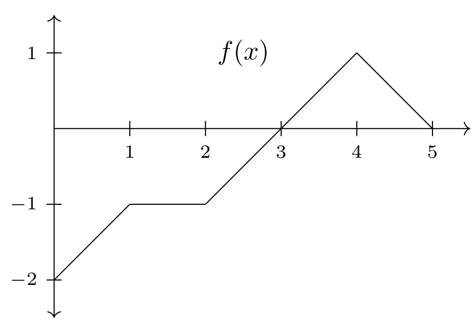
(d) $\displaystyle\int_1^6 |f(x)|\,dx$

113. Find the following antiderivatives. (Remember to put "$+c$")

(a) $\displaystyle\int 4^x\,dx$

(b) $\displaystyle\int \frac{1}{x}\,dx$

114. The figure below shows f. If $F' = f$ and $F(0) = 1$, find $F(b)$ for $b = 1, 2, 3, 4, 5$ and fill these values in the following table.



b	1	2	3	4	5
$F(b)$					

115. Compute the following antiderivatives

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(a) $\int \frac{1}{z}\,dz$

(b) $\int \cos(x) + \sin(x)\,dx$

(c) $\int 3^t + t^3\,dt$

(d) $\int \frac{1}{1+x^2}\,dx$

116. Compute the following antiderivatives:

(a) $\int y(y^2+2)^{10}\,dy$

(b) $\int \frac{e^x}{e^x-7}\,dx$

117. Compute the following derivatives:

(a) $\frac{d}{dx}\int_x^0 \ln(\cos(t))\,dt$

(b) $\frac{d}{dx}\int_1^{x^2} \cos(\tan(t))\,dt$

118. Find the solution to each initial value problem:

(a) $\frac{dP}{dt} = 5e^t,\;\; P(0) = -5$

(b) $\frac{dq}{dz} = 4 + \sin z,\;\; q(0) = 9$

119. At time, t, in seconds, your velocity, v, in meters/second, is given by

$$v(t) = t^2 - 3t + 5 \quad \text{for} \quad 0 \le t \le 6.$$

Use three partitions ($n = 3$) to estimate the distance traveled during this time using both right hand and left hand sums.

120. Calculate the following definite integrals and provide a family of antiderivatives for any indefinite integrals.

(a) $\int x\sqrt{x} + \frac{1}{x\sqrt{x}}\,dx$

(b) $\int_1^2 3^t \ln(3)\,dt$

121. A car is traveling at 40 ft/s and decelerates at a constant rate of 5 ft/s. How far does the car travel before coming to a complete stop?

122. Find the absolute area between the curves $f(x) = \cos(x)$ and $g(x) = \sin(x)$ on the interval $[0, \pi/2]$. (Hint: You must first find where these functions intersect and then determine which function is above and which is below on each segment of the interval.)

123. At time, t, in seconds, your velocity, v, in meters/second, is given by

$$v(t) = 1 + t^2 \quad \text{for} \quad 0 \le t \le 6.$$

Use $\Delta t = 2$ to estimate the distance traveled during this time. Find right hand and left hand estimates, and then average the two.

124. At time, t, in seconds, your velocity, v, in meters/second, is given by

$$v(t) = t^2 - 3t + 5 \quad \text{for} \quad 0 \le t \le 8.$$

Use Δ four partitions ($n = 4$) to estimate the distance traveled during this time using both right hand and left hand estimates.

Calculus Problem List

125. Find the difference between the right hand and left hand estimates for the area under $f(t)$ on the interval $a \leq x \leq b$ for n subdivisions.

$$f(t) = \sin t, \quad a = 0, \quad b = \pi/2, \quad n = 100$$

126. Use Figure 5.31 to find the values of

(a) $\int_a^b f(x)\,dx$	(b) $\int_b^c f(x)\,dx$
(c) $\int_a^c f(x)\,dx$	(d) $\int_a^c \lvert f(x)\rvert\,dx$

127. Find the average value of the function $f(x) = 4\sin(x)$ on the interval $[0, \pi]$

128. In 2005, the population of Mexico was growing at 1% a year. Assuming that this growth rate continues into the future and that t is in years since 2005, the Mexican population, P, in millions, will be given by

$$P = 103(a)^t.$$

Where $a = 1.01$. Predict the average population of Mexico between 2005 and 2055. (Consider a as a constant and keep your answers in terms of a, $\ln a$, etc.)

129. Find the area of the region between $y = x^{1/2}$ and $y = x^{1/3}$ for $0 \leq x \leq 1$.

130. Find the absolute area between the curves $f(x) = \cos(x)$ and $g(x) = \sin(x)$ on the interval $[0, \pi/2]$.

131. Suppose $\int_0^b f(x)\,dx = 12$. Calculate the integral $\int_{-b}^b f(x)\,dx$ if:
(a) $f(x)$ is an odd function (b) $f(x)$ is an even function

132. Use Figure 6.14 and the fact that $F(2) = 3$ to sketch the graph of $F(x)$. Label the values of at least four points.

133. Calculate the following definite integrals and provide a family of antiderivatives for any indefinite integrals.

(a) $\int x^9 - 3x^5 + 12x^3 - 2\,dx$	(f) $\int_1^2 3^t \ln(3)\,dt$
(b) $\int e^t - 7\,dt$	(g) $\int_0^{\pi/4} \cos(x) - \sin(x)\,dx$
(c) $\int y(y^2 - \frac{3}{y^2})\,dy$	(h) $\int_{-4}^4 (\sqrt{y} + 1)^2\,dy$
(d) $\int x\sqrt{x} + \frac{1}{x\sqrt{x}}\,dx$	(i) $\int t^2 + \frac{1}{t^2}\,dt$ $\int_1^2 \frac{1+y^2}{y}\,dy$
(e) $\int_0^3 \frac{1}{w} + w\,dw$	

134. Find the area enclosed by the curve $y = x^2(1 - x)^2$ and the x-axis.

135. A car is travelling at 40ft/s and decelerates at a constant rate of 5ft/s. How far does the car travel before coming to a complete stop?

136. A rock is thrown with a vertical velocity of 20 m/s from a height of 60m. What is the maximum height the rock reaches? How long does it take the rock to hit the ground? (You may assume that the acceleration of gravity is -10 m/s^2)

137. Calculate the following:

Calculus Problem List

(a) $\frac{d}{dx}\int_4^x \cos(x)\,dx$

(b) $\frac{d}{dx}\int_{-3}^{x^2} 5^x\,dx$

(c) $\frac{d}{dx}\int_{3x^2}^{\ln(x)} \tan(x)\,dx$

(d) $\frac{d}{dx}\int_{-15}^{x^2-2x} \frac{1}{x}\,dx$

Extras

138. Let $C(q)$ be the cost of producing q copies of a movie on blu–ray. See Figure 1 below.

   (a) What is the meaning of $C(0)$?

   (b) Describe in words how the marginal cost changes as the quantity produced increases.

   (c) Explain the concavity of the graph (in terms of economics).

   (d) Explain the economic significance (in terms of marginal cost) of the point at which the concavity changes.

   (e) Do you expect the graph of $C(q)$ to look like this for all types of products? (Hint: What if we sell the movies through itunes or some other electronic distribution?)

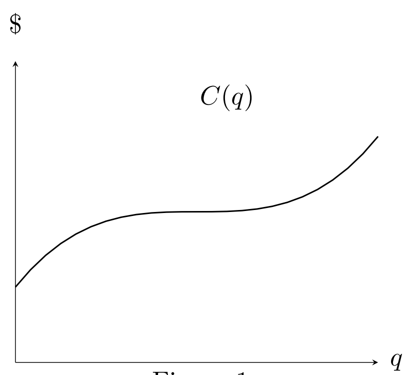

Figure 1

139. Using the cost and revenue graphs in Figure 2, sketch the following functions. Label the points q_1 and q_2.

   (a) Total Profit

   (b) Marginal Cost

   (c) Marginal Revenue

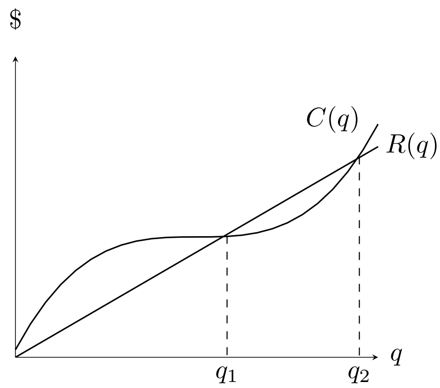

Figure 2

140. The total cost $C(q)$ of producing q goods is given by:

$$C(q) = 0.01q^3 - 0.6q^2 + 13q.$$

(a) What is the fixed cost?

(b) What is the maximum profit if each item is sold for $7 (assuming you sell everything you produce)?

(c) Suppose exactly 34 goods are produced. They all sell when the price is $7 each, but for each $1 increase in price, 2 fewer goods are sold. Should the price be raised, and if so by how much?